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SUPPRESSION OF IRREGULAR FREQUENCY EFFECTS IN FLUID–STRUCTURE INTERACTION PROBLEMS USING A COMBINED BOUNDARY INTEGRAL EQUATION METHOD

SIEW MING LAU AND GRANT E. HEARN

Department of Naval Architecture and Shipbuilding, University of Newcastle upon Tyne, Newcastle upon Tyne, U.K.

SUMMARY

It is well known that at certain discrete frequencies the conventional boundary integral equation formulation of free surface fluid–structure interaction analyses breaks down. At such 'irregular' frequencies the BIE method fails to provide either an acceptable or a unique solution. Having established the existence of irregular frequencies, a review of the different approaches adopted to remedy this problem is presented.

A very simple modification of the BIE method is also presented to eliminate the irregular frequency problem. The proposed procedure, designated the combined boundary integral equation method (CBIEM), can be categorized as a modified integral domain method. A description of the CBIEM formulation is presented and its ability to provide a unique solution at all frequencies is demonstrated. Predictions of 3D hydrodynamic reactive coefficients of added mass and fluid damping for a Series 60 hull form and an ellipsoid based on the CBIEM procedure are presented. These predictions are compared with results generated using conventional integral equation methods. The numerical studies demonstrate that the CBIEM is both a practical and effective method of suppressing irregular frequencies. In particular, the procedure is easy to implement in existing BIE computer codes with minimal additional computational effort.

KEY WORDS Combined boundary integral equation method Fluid–structure interaction Irregular frequencies

1. INTRODUCTION

The existence of irregular frequencies was first recognized by Lamb some 50 years ago in the field of acoustic radiation and scattering problems.[1] In the case of free surface water wave–body interaction problems, the occurrence of irregular frequencies was reported by John.[2,3] He showed that when the boundary value problem is reduced to an integral equation problem, with the wetted surface of the body representing the solution domain and the Green function acting as the kernel of the integral operator, the boundary integral equation does not admit a unique solution at some discrete frequencies. These frequencies correspond to a set of characteristic wavelengths for which the solution of the boundary value problem cannot be represented by a wave source integral formula. The breakdown generally occurs at wavelengths less than the characteristic linear dimension of the structure. In the case of ships, these wavelengths are often much smaller than the ship length.

Hitherto, studies of the irregular frequency phenomenon have been concerned with the zero speed problem or the forward speed problem solved as a zero speed equivalent problem.[4] Little is known about the existence and influence of the irregular frequencies in the forward speed integral

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equation formulation. The theoretical investigation of the forward speed problem is considerably complicated because of the necessary inclusion of an additional line integral and the non-symmetric properties of the oscillating and translating wave source.

In this paper we shall first review the existence of irregular frequencies in the 3D zero speed integral equation and then show how they are related to the interior Dirichlet problem. Various methods of modifying the integral equation to overcome the irregular frequency difficulty are then discussed. In particular we shall introduce a combined boundary integral equation method which is uniquely solvable for all frequencies. The equation is derived from supplementing the exterior Fredholm integral equation of the second kind with an integral equation of the first kind in the interior domain of the body. The modified method is then applied to provide numerical results for the hydrodynamic added mass and fluid damping coefficients of free floating structures. These results are also presented with results generated using a standard source integral equation and the direct Green function integral methods.

2. THE EXISTENCE OF IRREGULAR FREQUENCIES

For a free floating body in waves the hydrodynamic boundary value problem can be reduced to an integral equation with the solution domain corresponding to the wetted surface of the body. The integral equation is obtained either through direct application of Green's theorem or the use of a source distribution. The source formulation is often referred to as the 'indirect method', since the velocity potential is determined after the source distribution strengths have been determined. It can be shown, as far as the study of irregular frequencies is concerned, that the two approaches have an identical set of characteristic frequencies at which both formulations fail. It is therefore necessary to establish whether the integral equation has a solution at the irregular frequency, and if a solution does exist whether or not it is unique.

The integral equation representing the source strength distribution is given by

$$2\pi\sigma(p) + \iint \sigma(q)\frac{\partial}{\partial n_p} G(p, q)\,\mathrm{d}s = \frac{\partial}{\partial n_p}\phi(p), \tag{1}$$

where p and q are the 'source' and 'field' points respectively. On the hull wetted surface, the right-hand side of equation (1) can be equated to V_n, the normal component of the fluid velocity. This integral equation is the governing equation of the so called external Neumann problem, with the normal derivative of the Green function acting as the kernel operator. The inhomogeneous equation is also known as a Fredholm integral equation of the second kind. Such equations are known to break down at certain discrete frequencies. In order to demonstrate this we must resort to the Fredholm integral theorem.

The Fredholm integral can be written in the general form

$$f(p) + \mu\int K(p, q) f(q)\,\mathrm{d}s = W(p), \tag{2a}$$

with the associated homogeneous equation written as

$$f_0(p) + \mu\int K(p, q) f_0(q)\,\mathrm{d}s = 0. \tag{2b}$$

The adjoint homogeneous equation is defined by

$$g_0(p) + \mu\int K^*(p, q) g_0(q)\,\mathrm{d}s = 0, \tag{2c}$$

where the superscript $*$ denotes a complex conjugate. Then, according to the Fredholm integral theorem, if the homogeneous equation (2b) has a non-trivial solution, the integral equation (2a) will have a solution if and only if $W(p)$ is orthogonal to every solution of the adjoint homogeneous equation, that is

$$\int g_0^*(p)\, W(p)\, \mathrm{d}s = 0. \tag{3}$$

Furthermore, even if equation (3) is satisfied, the solution to equation (2a) cannot be uniquely determined since any multiple of $f_0(p)$ may be added to the particular solution. The parameter μ for which the homogeneous equation (2b) has a non-trivial solution is called a 'characteristic value' of the kernel $K(p, q)$, otherwise μ is said to have a 'regular value'.

The results of the Fredholm integral theorem are now applied to the source integral equation (1). Clearly, comparing equations (1) and (2a), we note for our particular formulation that

$$\mu = 1, \quad f(p) = \sigma(p), \quad K(p, q) = \frac{1}{2\pi}\frac{\partial}{\partial n_p} G(p, q), \quad W(p) = \frac{1}{2\pi}\frac{\partial}{\partial n_p}\phi(p).$$

Consequently, the compatibility equation (3) becomes

$$\iint \sigma_0^*(p) \frac{\partial}{\partial n_p}\phi(p)\, \mathrm{d}s = 0, \tag{4}$$

where σ_0^* is the non-trivial solution of the corresponding adjoint homogeneous equation written as

$$2\pi\sigma_0(p) + \iint \sigma_0(q) \frac{\partial}{\partial n_q} G^*(q, p)\, \mathrm{d}s = 0. \tag{5}$$

Since the Green function $G(p, q)$ is a function of the wave number k_0, then for each value of k_0 there is a set of characteristic values of $(\partial/\partial n_p) G(p, q)$. The wave number k_0, subject to $\mu = 1$ in its set of characteristic values, is called the 'critical wave number'. For this wave number value the source integral equation given by equation (1) has either no solution or the solution is not unique.

In the external Neumann problem the value of $W(p)$ or $(\partial/\partial n_p)\phi(p)$ is specified on the surface of the body boundary. Therefore for an arbitrary shaped body, except for some very special velocity distributions, it is unlikely that the compatibility equation (4) will be satisfied because of the arbitrary nature of $W(p)$. Therefore for certain characteristic wave numbers the source formulation will have no solution and the boundary value problem cannot be represented by the source integral equation formulation (1).

Next we demonstrate how the irregular frequencies or characteristic values of the exterior Neumann problem are related to the eigensolutions of the interior Dirichlet problem. The interior Dirichlet problem is fictitious in that its solution has no physical relationship with the velocity potential distribution of the exterior Neumann problem. Consider a point $\hat{p}$ located in the interior domain of the body whose inner velocity potential is denoted by $\hat{\phi}$. By definition $\hat{\phi}$ satisfies Laplace's equation and the same free surface condition as the exterior potential. Using Green's second identity and letting the field point $\hat{p}$ approach the wetted surface boundary S_0 from the inside, so that $\hat{p}$ tends to p, the integral equation describing the interior problem can be written as

$$2\pi\hat{\phi}(p) - \iint \hat{\phi}(q) \frac{\partial}{\partial n_q} G(p, q)\, \mathrm{d}s = -\iint \frac{\partial}{\partial n_q}\hat{\phi}(q) G(p, q)\, \mathrm{d}s. \tag{6}$$

The classification of this integral equation depends upon the boundary conditions specified on S_0. Thus equation (6) becomes an integral equation of the first kind if the velocity potential is specified on the boundary, and equation (6) is an integral equation of the second kind if the fluid normal velocity is specified on S_0. To obtain an integral equation of the second kind with the velocity potential specified on the boundary surface, i.e. a Dirichlet problem, we differentiate equation (6) with respect to the normal vector at the point p on S_0, that is

$$2\pi \frac{\partial}{\partial n_p} \hat{\phi}(p) + \iint \frac{\partial}{\partial n_q} \hat{\phi}(q) \frac{\partial}{\partial n_p} G(p, q)\,\mathrm{d}s = \frac{\partial}{\partial n_p} \iint \frac{\partial}{\partial n_q} G(p, q) \hat{\phi}(q)\,\mathrm{d}s. \tag{7}$$

The right-hand side of equation (7) is known in the Dirichlet problem, since $\hat{\phi}$ is prescribed on S_0. Comparing the kernel of equation (7) with that of equation (1), representing the exterior Neumann problem, it is clear that the kernels are identical. This means that whenever the homogeneous equation corresponding to equation (7) has a non-trivial solution, the exterior homogeneous Neumann problem associated with equation (1) also has a non-trivial solution. Thus equation (1) has a solution if and only if the compatibility equation (4) is satisfied. In other words, the eigensolution of the interior Dirichlet problem corresponds to the irregular frequencies of the exterior Neumann problem.

An alternative formulation of the exterior problem is provided by directly appealing to Green's second identity. In this case the integral equation to be solved is given by

$$2\pi\phi(p) + \iint \phi(q) \frac{\partial}{\partial n_q} G(p, q)\,\mathrm{d}s = \iint \frac{\partial}{\partial n_q} \phi(q) G(p, q)\,\mathrm{d}s, \tag{8}$$

and the corresponding adjoint homogeneous equation is

$$2\pi g(p) + \iint g(q) \frac{\partial}{\partial n_p} G^*(q, p)\,\mathrm{d}s = 0. \tag{9}$$

Thus the compatibility equation (3) now assumes the form

$$\iint g^*(q)\left(\iint \frac{\partial}{\partial n_q} \phi(q) G(p, q)\,\mathrm{d}s \right)\mathrm{d}s = 0, \tag{10}$$

where $g^*(q)$ is the complex conjugate solution of the adjoint homogeneous equation (9). One should note that whereas the normal derivative in equation (8) is now taken with respect to the dummy or field point variable q, the normal derivative of equation (1) is taken with respect to the free or source point variable p.

To prove that the compatibility equation (10) can indeed be satisfied in the exterior Neumann problem, we return to the interior Dirichlet problem given by equation (6). For the case of the homogeneous Dirichlet problem, equation (6) reduces to

$$\iint \frac{\partial}{\partial n_q} \hat{\phi}(q) G(p, q)\,\mathrm{d}s = 0. \tag{11}$$

If we now take the complex conjugate of the adjoint homogeneous equation (9) and compare it with the homogeneous form of the interior Dirichlet problem given by equation (7), the identity

$$g^*(q) = \frac{\partial}{\partial n_q} \hat{\phi}(q) \tag{12}$$

can be established. Changing the order of integration in equation (10) and making use of the identity expressed in equation (12) yields

$$\iint \frac{\partial}{\partial n_q} \phi(q)\left(\iint \frac{\partial}{\partial n_q} \hat{\phi}(q) G(p, q)\,\mathrm{d}s \right)\mathrm{d}s = 0. \tag{13}$$

The inner integral of equation (13) is equal to equation (11) and thus the integrand of equation (13) vanishes identically. That is, the compatibility equation (10) is satisfied for all conjugate solutions $g^*(q)$ of the adjoint homogeneous equation (9). This result is sufficient to guarantee a solution to the integral equation (8), though the solution is not unique at the critical wave numbers. On the other hand the integral equation formulation based on the source distribution has no solution at the eigenfrequencies since the compatibility equation (4) is unlikely to be satisfied for an arbitrary shaped body.

3. METHODS OF REMOVING THE IRREGULAR FREQUENCIES

Although the source integral equation (1) and the direct Green function integral equation (8) are not solvable at certain discrete sets of wave frequencies, the effect of irregular frequencies upon the prediction of the hydrodynamic coefficients is quite detrimental. This is because the effect of each irregular frequency can often spread over quite a wide frequency band centred on their exact location. In particular, the integral equation becomes ill-conditioned in the vicinity of these frequencies. The error due to the ill-conditioning can be further aggravated by small perturbations in the formulation due to either numerical inaccuracy arising in the evaluation of the Green function or to poor modelling of the body wetted surface. Numerical results presented by Breit[5] have shown that the frequency band of the irregular frequencies can generally be reduced if the surface representation is improved by refining the hull mesh or by using higher-order representations. There exist many methods of modifying the integral equation to ensure a unique solution at all frequencies. The choice of methods has to be considered in terms of their effectiveness, the computational penalties and the ease of implementation. Broadly speaking, the modifications that have been suggested hitherto can be grouped into two categories:

(a) modification of the integral operator
(b) modification of the domain of the integral operator.

In the second category the domain of the integral operator is enlarged, whereas in the first category the integral operator is modified on the same domain. The second category is sometimes also referred to as the 'extended boundary condition' method.

3.1. Modifying the integral operator

The procedure for modifying the integral operator was first suggested by Ursell.[6] The method involves adding a series of multipole solutions of the governing equation to the associated Green function. In the field of acoustics, Ursell[7] and Jones[8] showed that a suitably modified Green function kernel of the integral equation always produced a unique solution for all frequencies. The number of multipoles added determined the range of wave frequencies which were free of irregular frequencies. The choice of the constants associated with the multipoles has been studied by Kleinman and Roach[9] who provide some criteria regarding the optimal selection of these constants. The multipole method in the water wave problem has the advantage of low computation time, but unfortunately it is not always convenient for bodies with complicated shapes and does not generalize to full 3D analyses.

In water wave problems Sayer and Ursell[10] illustrated that the irregular frequencies can be removed by augmenting the Green function with an additional wave singularity inside the body. The modified Green function G' is written as

$$G'(p, q) = G(p, q) + AG(p, 0)G(q, 0), \tag{14}$$

where $G(p, q)$ is the original Green function and A is a constant. The additional wave singularity,

$G(p, 0)$ and $G(q, 0)$, is placed at the origin and its influence is evaluated at points p and q respectively. Similar approaches have been adopted by Ogilvie and Shin,[11] Adachi and Ohmatsu,[12] Sayer[13] and Wu and Price[14] in the study of two-dimensional wave radiation and diffraction problems.

Martin and Ursell[15] provided an alternative method of solving the boundary value problem called the null-field equation approach. The fundamental difference between the null-field method and the usual integral equation methods is that the null-field governing equations do not represent an integral equation of the second kind. In fact the method provides an infinite set of moment-like equations, called null-field equations. Martin and Ursell were able to show that these equations are uniquely solvable for all frequencies.

Another alternative method, proposed by Burton and Miller[16] for the acoustic scattering problem, is to exploit the different locations of the eigenfrequencies in the first and second kind integral equations. In particular they show that a linear combination of two such equations for the exterior Neumann problem will always provide a unique solution at all frequencies. This equation can be written as

$$2\pi\hat{\phi}(p) + \iint \hat{\phi}(q)\frac{\partial}{\partial n_q}\left(1 + \alpha\frac{\partial}{\partial n_p}\right)G(p, q)\,\mathrm{d}s$$
$$= -2\pi\alpha\frac{\partial}{\partial n_p}\phi(p) + \iint \frac{\partial}{\partial n_q}\phi(q)\left(1 + \alpha\frac{\partial}{\partial n_p}\right)G(p, q)\,\mathrm{d}s, \tag{15}$$

where α is a purely imaginary constant of the form $\alpha = \mathrm{i}a$, with a a real number. Recently, Sclavounos and Lee[17] adopted this method in the solution of the 2D free surface radiation and diffraction problems. They suggested that optimum performance of the method was associated with a value of a in the range 0·2–0·3.

The major drawback of the Burton and Miller procedure is the evaluation of the double normal derivative of the Green function at all points on the body surface. This is likely to incur a substantial amount of extra computing effort and core memory storage.

3.2. Modifying the domain of the integral operator

A well known method of modifying the domain of the integral equation to eliminate the irregular frequencies is the so called 'capping' procedure. The method is based on the idea that the irregular frequencies are associated with a resonant wave phenomenon in the corresponding interior problem. For a harmonic oscillating body one can imagine a kind of fluid sloshing inside the body, and thus the placing of a lid or cap on the interior free surface might suppress resonant responses. The cap is formed by extending the source or the dipole distribution onto the interior free surface and imposing a rigid wall condition on it. Ohmatsu[18] proved that such a procedure is indeed mathematically justifiable and confirmed its viability numerically. Recently, Donati[19] carried out extensive numerical studies on the performance of the capping procedure. The effects of different section shapes and the number of facets used to represent the cap when determining the 2D hydrodynamic coefficients were investigated. Donati concluded that, in general, two facets would suffice to overcome the effects of the first and second irregular frequencies. At higher frequencies a minimum of eight to twelve facets was required, with the actual number depending upon the section shape and the particular hydrodynamic coefficients being considered. In particular, Donati found that using too many facets on the cap could have detrimental effects on the values of the hydrodynamic coefficients clear of the irregular frequencies. This unexpected drawback places some uncertainty on the general reliability of the capping procedure for practical use. In three-dimensional problems capping also has the disadvantage of incurring large additional computation costs due to the 3D discretization of the interior free surface.

3.3. The combined boundary integral equation method (CBIEM)

The proposed combined boundary integral equation method, abbreviated to CBIEM, involves a modification of the domain of the integral operator. The basic method was first suggested by Schenck[20] in the field of acoustics. The method can also be said to follow the ideas of Burton and Miller,[16] namely exploiting the different locations of the eigenfrequencies in the first and second kind integral equations.

In fact the main difference in the CBIEM is that the supplementary integral equation of the first kind is obtained through extending the field point of the integral equation into the interior domain of the body rather than by taking its normal derivative on the body surface. On its own the interior first kind integral equation has some undesirable computational characteristics, the principal objection being that all the influence matrix coefficients become smaller as the body surface division is refined. This is in direct contrast to the second kind integral equation for which the diagonal terms of the influence matrix tend to a constant and only the off-diagonal terms decrease. However, the observed shortcomings can be overcome by combining the interior first kind problem with the exterior second kind equation based on the body wetted surface. Since these equations are not inconsistent, they form an overdetermined system whose solution will be unique. The uniqueness of the solution of the CBIEM will be proved by considering the interior Dirichlet problem.

3.4. Formulation and uniqueness of the CBIEM

From Green's theorem, the velocity potential ϕ of the exterior Neumann problem can be written as

$$\iint \phi(q)\frac{\partial}{\partial n_q} G(p,q)\,\mathrm{d}s - \frac{\partial}{\partial n_q}\phi(q)G(p,q)\,\mathrm{d}s = \begin{cases} 4\pi\phi(p), & p \text{ in } D, & \text{(16a)} \\ 2\pi\phi(p), & p_0 \text{ on } S_0, & \text{(16b)} \\ 0, & p \text{ in } \hat{D}, & \text{(16c)} \end{cases}$$

where D and $\hat{D}$ are the exterior and interior domains of the body respectively. The body wetted surface S_0 is assumed to be smooth. The second kind integral equation (16b) represents a dipole distribution of strength ϕ and a source distribution of strength $\partial\phi/\partial n_q$ on the surface S_0. For a point p lying in the interior domain $\hat{D}$ of the body, equation (16c) provides an integral equation of the first kind. By itself the integral equation of the first kind is not uniquely solvable at the eigenfrequencies, but then these frequencies do occur at different locations to those of the second kind integral equation. This property suggests that if equations (16b) and (16c) are combined and solved together, then the solution will always be unique. This indeed turns out to be the case since only one of the many solutions at the eigenfrequencies of the exterior Neumann problem satisfies equation (16c).

Let ϕ_1 be the solution of equation (16b) that also satisfies equation (16c) at the irregular frequencies; then

$$2\pi\phi_1(p) + \iint \phi_1(q)\frac{\partial}{\partial n_q} G(p,q)\,\mathrm{d}s = \iint G(p,q)\,V_n(q)\,\mathrm{d}s \tag{17}$$

and

$$\iint \phi_1(q) \frac{\partial}{\partial n_q} G(\hat{p}, q)\,\mathrm{d}s = \iint G(\hat{p}, q) V_n(q)\,\mathrm{d}s, \tag{18}$$

where $\hat{p}$ is a point in $\hat{D}$. Also let ϕ_0 be the non-trivial solution of the corresponding homogeneous equation; then

$$2\pi\phi_0(p) + \iint \phi_0(q) \frac{\partial}{\partial n_q} G(p, q)\,\mathrm{d}s = 0. \tag{19}$$

Then any other solutions that satisfy equation (17) can be written as

$$\phi_2 = \phi_1 + A\phi_0, \tag{20}$$

where A is a constant. If we assume that the solutions given by equation (20) also satisfy equation (18), it follows that

$$\iint (\phi_1 + A\phi_0) \frac{\partial}{\partial n_q} G(\hat{p}, q)\,\mathrm{d}s = \iint G(\hat{p}, q) V_n(q)\,\mathrm{d}s. \tag{21}$$

However, ϕ_1 satisfies equation (18) and hence equation (21) reduces to

$$A \iint \phi_0(q) \frac{\partial}{\partial n_q} G(\hat{p}, q)\,\mathrm{d}s = 0. \tag{22}$$

Consequently equation (22) suggests that either $A = 0$ or the integral itself is equal to zero. If $A = 0$ then we have $\phi_2 = \phi_1$, which implies that the integral equation (18) only admits one solution at the eigenfrequencies of equation (17). This can be proved mathematically by again resorting to the interior formulation of the velocity potential.

Let us denote the interior potential by $\hat{\phi}(p)$. This potential can also be represented by a distribution of dipoles of strength γ over the boundary surface S_0, such that

$$\hat{\phi}(\hat{p}) = \iint \gamma(q) \frac{\partial}{\partial n_q} G(\hat{p}, q)\,\mathrm{d}s. \tag{23}$$

When the interior point $\hat{p}$ approaches the boundary surface S_0, then

$$\hat{\phi}(p) = 2\pi\gamma(p) + \iint \gamma(q) \frac{\partial}{\partial n_q} G(p, q)\,\mathrm{d}s. \tag{24}$$

Comparing the kernel of equation (24) with that of equation (17) reveals that they are identical. This implies that if the homogeneous equation associated with equation (24) has a non-trivial solution, then the homogeneous equation corresponding to equation (17) will also have a non-trivial solution. Therefore at the eigenfrequencies, the eigensolution of the interior Dirichlet problem will be given by the right-hand side of equation (23). This therefore suggests that

$$\iint \gamma(q) \frac{\partial}{\partial n_q} G(\hat{p}, q)\,\mathrm{d}s \neq 0$$

and thus implies that $A = 0$ in equation (22). This completes the proof of the uniqueness of the combined boundary integral equations.

4. IMPLEMENTATION OF THE CBIEM

The two integral equations of the first and second kind to be solved in the CBIEM are

$$2\pi\phi(p) + \iint_{S_0} \phi(q)\frac{\partial}{\partial n_q} G(p,q)\,\mathrm{d}s = \iint_{S_0} V_n(q)G(p,q)\,\mathrm{d}s, \tag{25}$$

$$\iint_{S_0} \phi(q)\frac{\partial}{\partial n_q} G(\hat{p},q)\,\mathrm{d}s = \iint_{S_0} V_n(q)G(\hat{p},q)\,\mathrm{d}s, \tag{26}$$

where V_n is the normal fluid velocity component on the hull surface.

The combined integral equation (25) and (26) can be solved numerically by the usual method of surface discretization. The resultant non-square overdetermined $(N + N')$ by N matrix can be transformed to an N by N system by the least-squares orthonormalizing procedure.[21] The transformed matrix coefficients D_{ij} are given by

$$D_{ij} = \sum_{k=1}^{N+N'} d_{ki}d_{kj}, \quad i,j = 1 \text{ to } N,$$

where N is the total number of nodes on the body surface and N' is the number of interior points used in association with equation (26). The coefficients d_{ki} and d_{kj} are the elements of the non-square matrix.

The main advantage of the CBIEM is its simplicity and ease of implementation. An existing computer program based on the direct Green integral can be easily modified to allow for the extension of field points into the interior domain of the body. If the boundary value problem has been formulated as a source strength distribution problem, then one more additional change will be necessary, namely the position at which the normal derivative of the Green function is evaluated. In the source strength method the derivatives are evaluated at the field points, whereas in the direct method they are evaluated at the source points.

The position of the interior points $\hat{p}$ are arbitrary so long as they do not coincide with the nodal points of the interior eigensolution. For an arbitrary shaped body the nodal points of the interior eigensolution are not known *a priori*, and moreover they increase in number as the wave frequency increases and also become more closely packed together. Consequently the number of interior points required in the application of the CBIEM has to be increased to avoid these nodal points and this can lead to greater computational costs. This therefore places a limit on the effectiveness of the CBIEM for frequencies beyond some threshold for which the performance will be impaired. In order to alleviate this deficiency, Ohmatsu[22] suggested adding an extra equation obtained by taking the x-derivative of equation (26) into the overdetermined system. The reason for doing this is that the x-derivatives of the interior potential cannot be zero, even at the nodal points. We have not implemented this in our 3D program because we are primarily interested in the frequency range within the first few irregular frequencies and therefore just adding extra interior points is considered to be more cost effective. The advantage provided by incorporating the x-derivative of the interior potential has to be balanced against the extra computational effort required in evaluating the second-order derivatives of the Green function in equation (26).

For a body with geometric symmetry, the boundary value problems have solutions which are symmetric and antisymmetric. In the case of a ship, the symmetric hydrodynamic quantities are associated with the surge, heave and pitch motions and the antisymmetric quantities relate to the sway, roll and yaw motions. Therefore the influence matrix coefficients from the discretized interior integral equation (26) are processed to form the sum and the difference of the port and the starboard side contributions, and these are used to determine the symmetric and the antisymmetric modes respectively. If the interior point $\hat{p}$ is located on the symmetric centreplane of the ship, the matrix coefficients of the antisymmetric mode will have no contribution from the supplementary interior integral equation. For this reason the interior points should be placed off the centreplane.

4.1. *Numerical results and discussions*

Numerical predictions of the added mass and the fluid damping coefficients based on three different integral formulations are presented for comparison. The three boundary integral equation methods used are the source distribution method, equation (1), the direct boundary integral equation method, equation (8), and the CBIEM, equations (25) and (26). The structures used in the applications are a Series 60 ship, with a block coefficient 0·7, and a half submerged ellipsoid. The Series 60 model and the ellipsoid are discretized into 216 and 220 facets respectively; see Figures 1 and 2. The principal dimensions of both models are given in the Appendix.

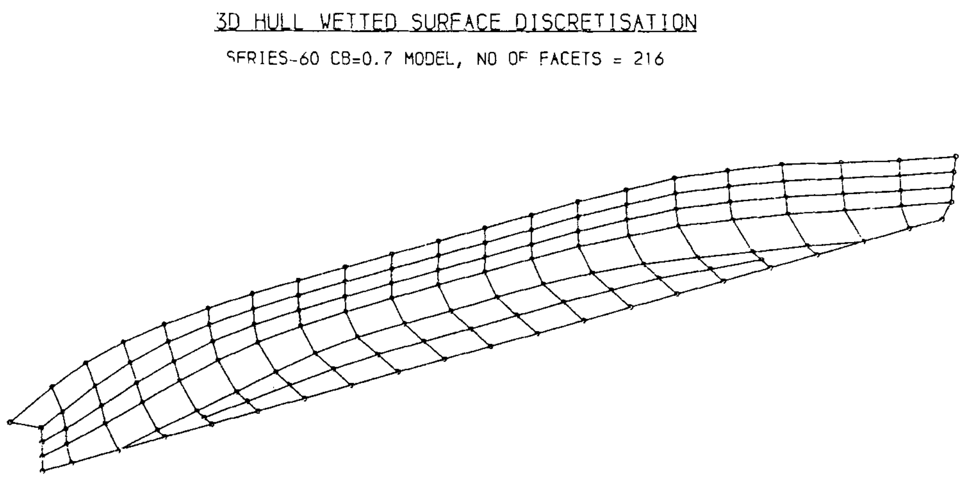

Figure 1. 3D discretized hull surface of a Series 60 model; 216 facets; $C_b = 0{\cdot}7$

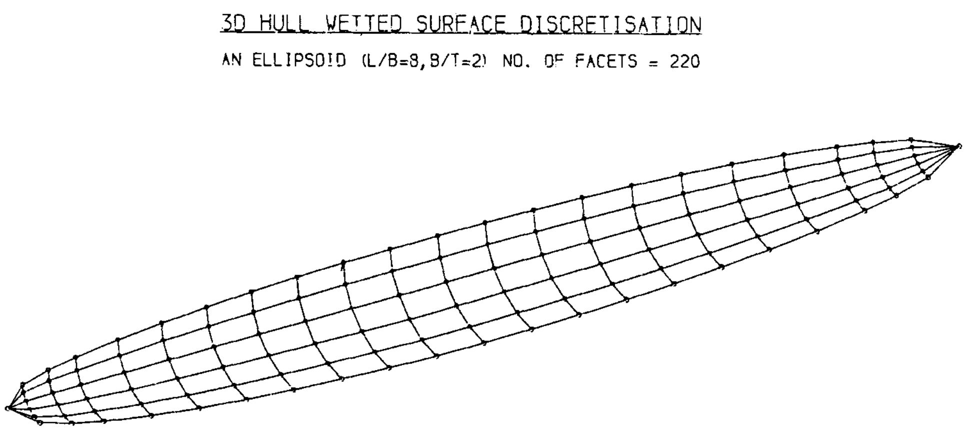

Figure 2. 3D discretized hull surface of an ellipsoid; 220 facets; L/B = 8, B/T = 2

Figures 3–10 provide the hydrodynamic coefficients of the Series 60 model. These coefficients are computed on the basis of the forward speed Green function of Lau[23] applied for a vanishingly low Froude number of $F = 0{\cdot}001$. The first irregular frequency occurs when the non-dimensional frequency $f(\omega_e\sqrt{L/g})$ approaches 5·15 for which both the source and the direct methods fail. The errors in the predicted hydrodynamic coefficients are substantial within the vicinity of the irregular frequency and their effect spreads over quite a wide frequency band owing to the ill-conditioning of the influence coefficient matrices. The 'spread' of the influence of the irregular frequency is more severe on the fluid damping coefficients than on the predicted added masses. In the case of the cross-coupling hydrodynamic coefficients the effect of the irregular frequency is sufficient to destroy the required Timman–Newman[24] symmetry relationships. However, the computed results based on the CBIEM with 10 interior points do not exhibit any irregular frequency effects. The use of the CBIEM also restores the expected symmetric relationship in the cross-coupling hydrodynamic coefficients at and near the first irregular frequency.

As commented on earlier, Donati[19] reported that the use of too many interior facets in the capping procedure can have a detrimental effect upon the predicted results. In order to investigate whether the number of interior points used in the CBIEM can have a similar adverse effect upon

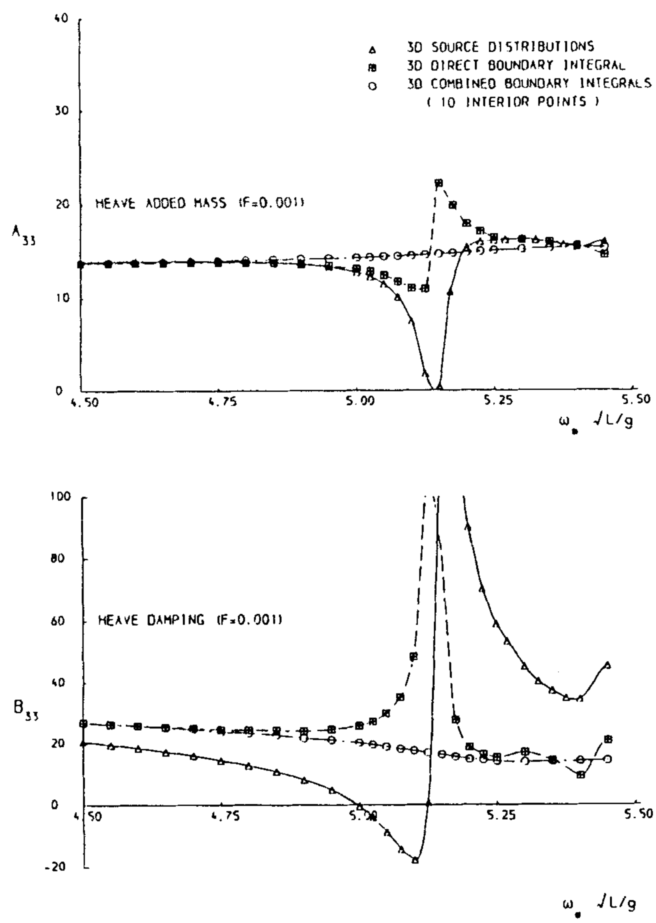

Figure 3. Heave added mass and damping coefficients

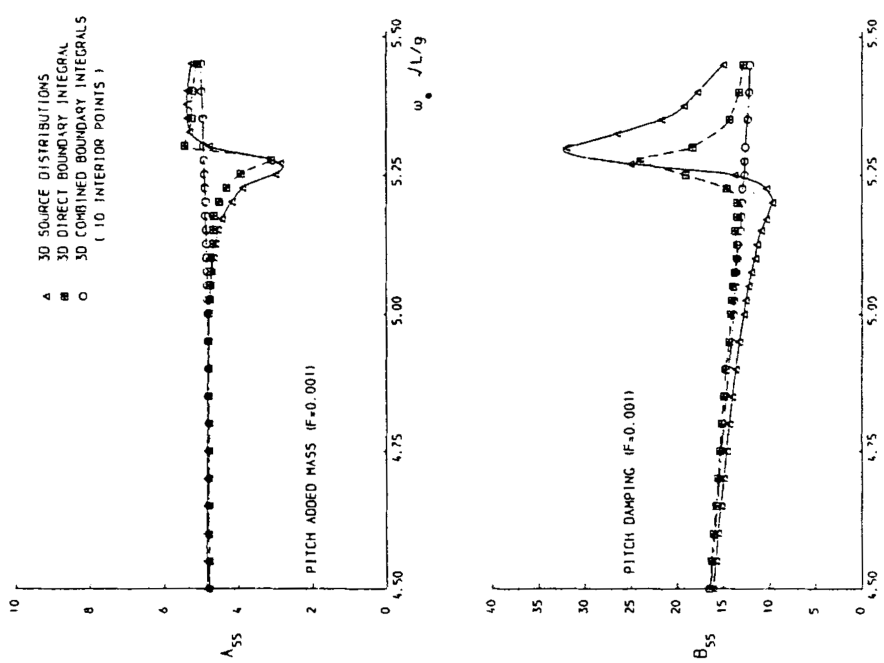

Figure 4. Pitch added mass and damping coefficients

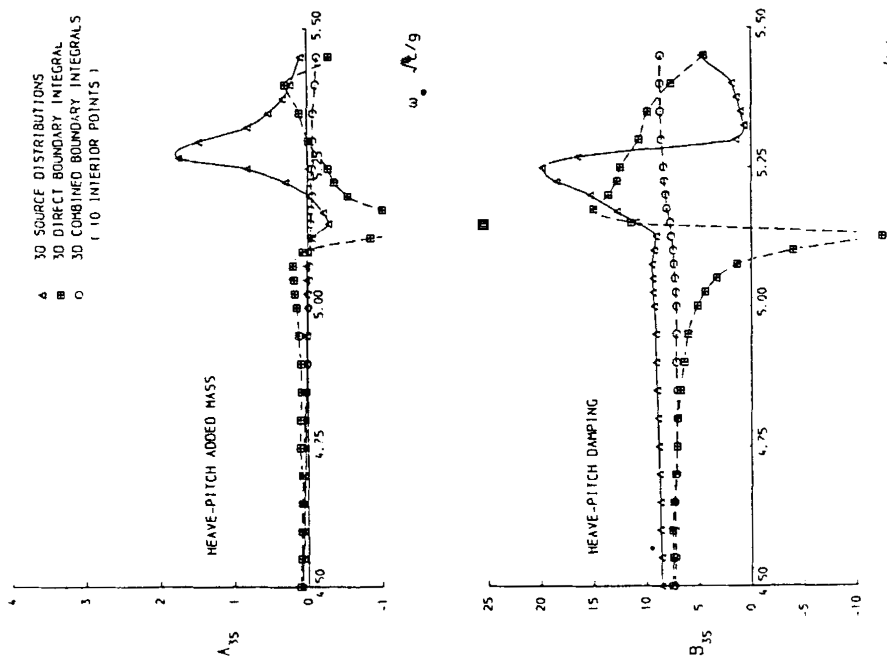

Figure 5. Pitch-induced heave added mass and damping coefficients

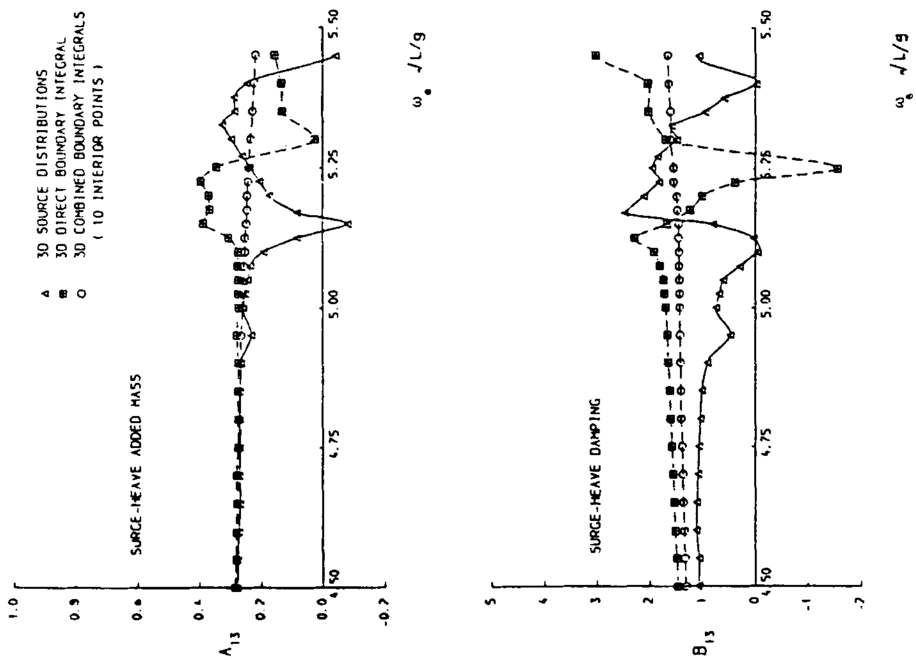

Figure 7. Heave-induced surge added mass and damping coefficients

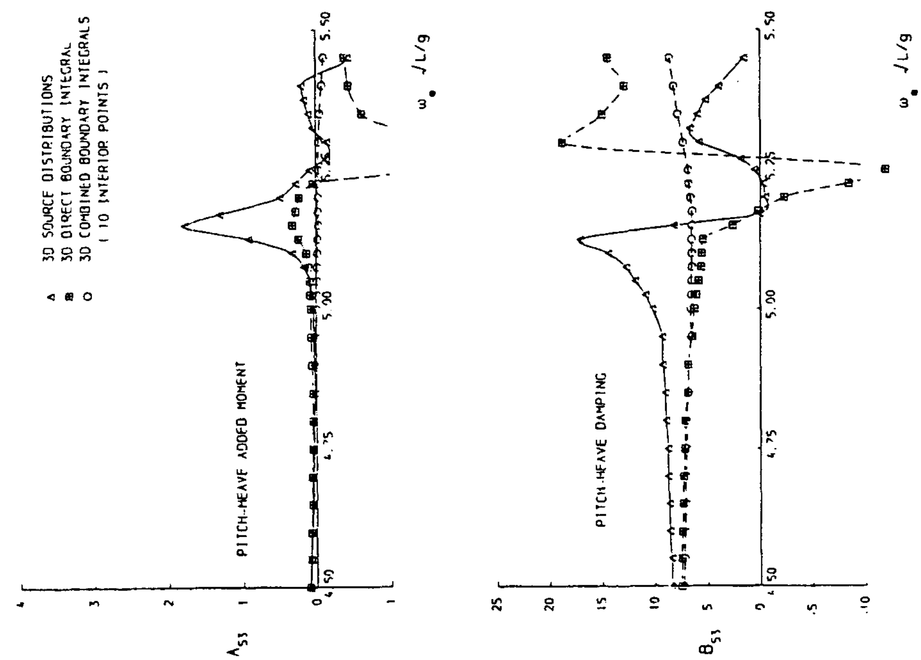

Figure 6. Heave-induced pitch added mass and damping coefficients

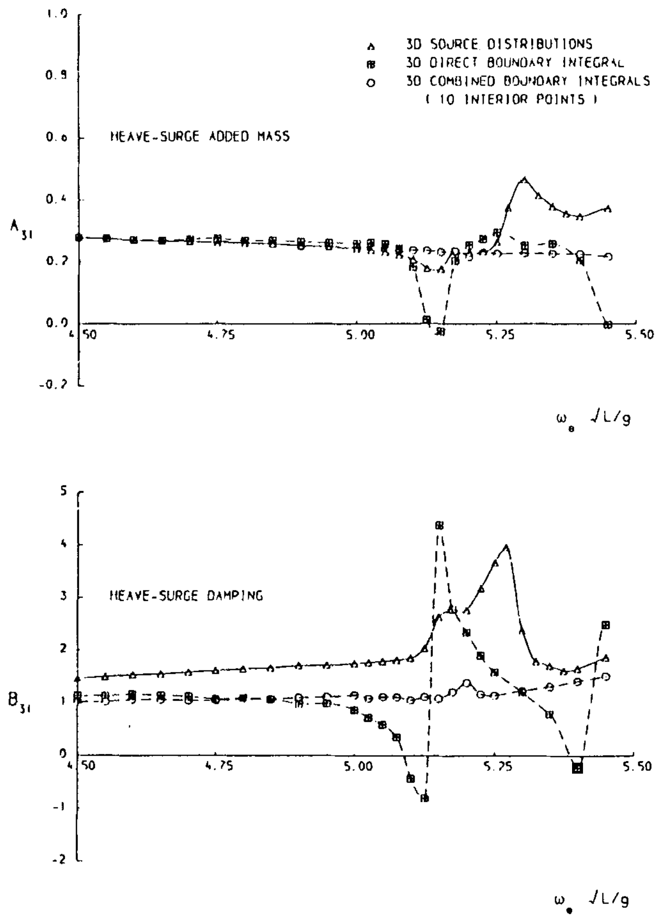

Figure 8. Surge-induced heave added mass and damping coefficients

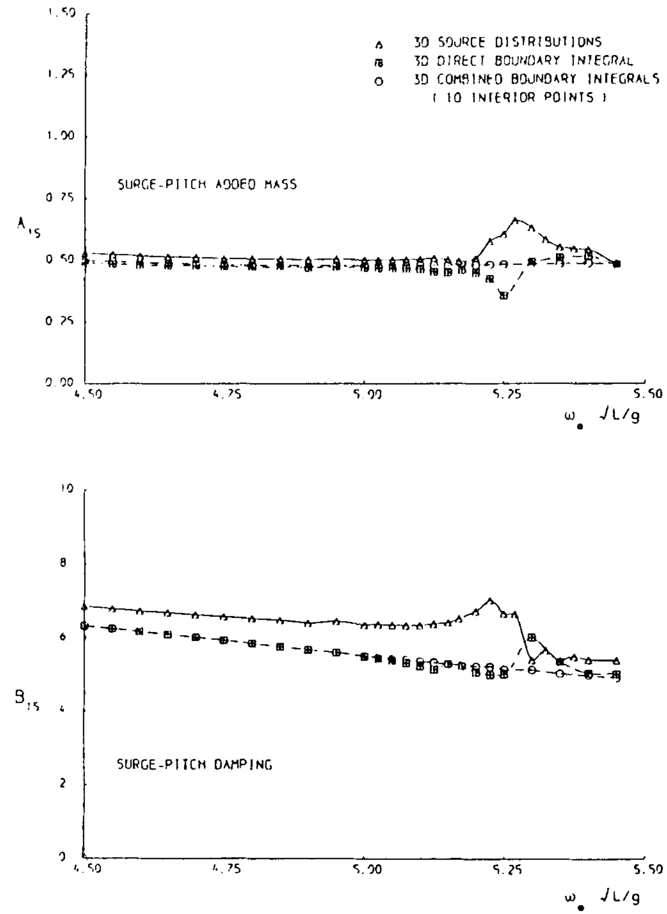

Figure 9. Pitch-induced surge added mass and damping coefficients

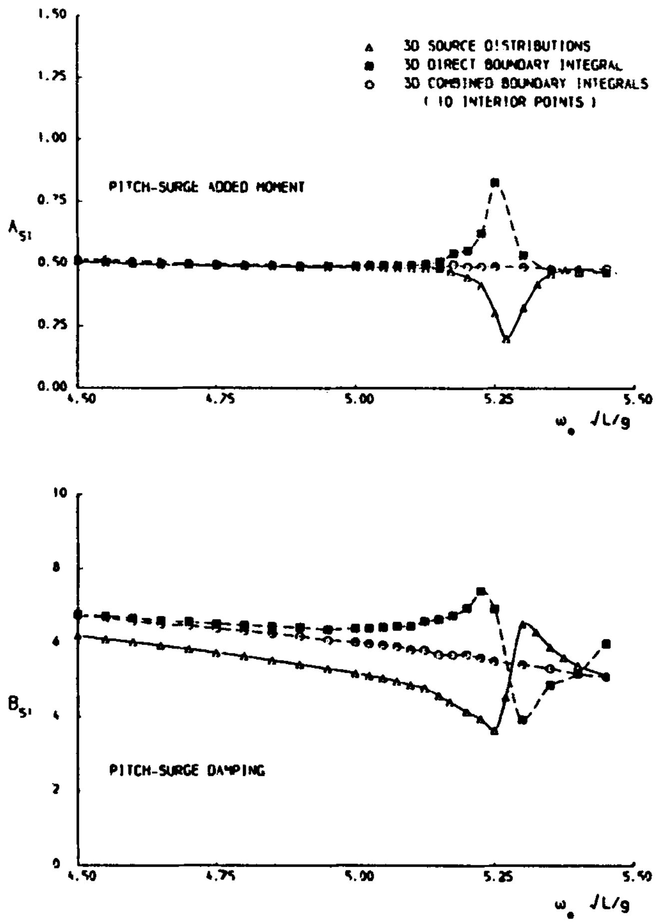

Figure 10. Surge-induced pitch added mass and damping coefficients

the hydrodynamic coefficients, the calculations were repeated for a varying number of interior points. Figures 11 and 12 display the results at the wave frequency $f = 5{\cdot}25$ with N', the number of interior points, ranging from 0 to 20, where $N' = 0$ corresponds to the direct boundary integral method. These figures show that at the selected frequency a minimum of 10 interior points are needed to ensure consistent results for the different hydrodynamic coefficients considered. Some hydrodynamic coefficients, such as the pure surge, the pure pitch and the heave–pitch added masses, require as few as 4 interior points to eliminate the first irregular frequency. When N' is less than 4 the CBIEM predicted results are actually worse than either the source or direct methods which correspond to $N' = 0$.

The additional computational effort required, in terms of CPU time, for the CBIEM in comparison with the direct method is approximately given by N'/N, where N is the total number of surface facets used to represent the wetted surface of the structure. If the model has one plane of symmetry, this figure is to be doubled. For the Series 60 model, with N equal to 216 and the maximum N' value equal to 20, it is immediately apparent that the additional computational cost incurred by the CBIEM over that of the direct method is not excessive. Table I provides the

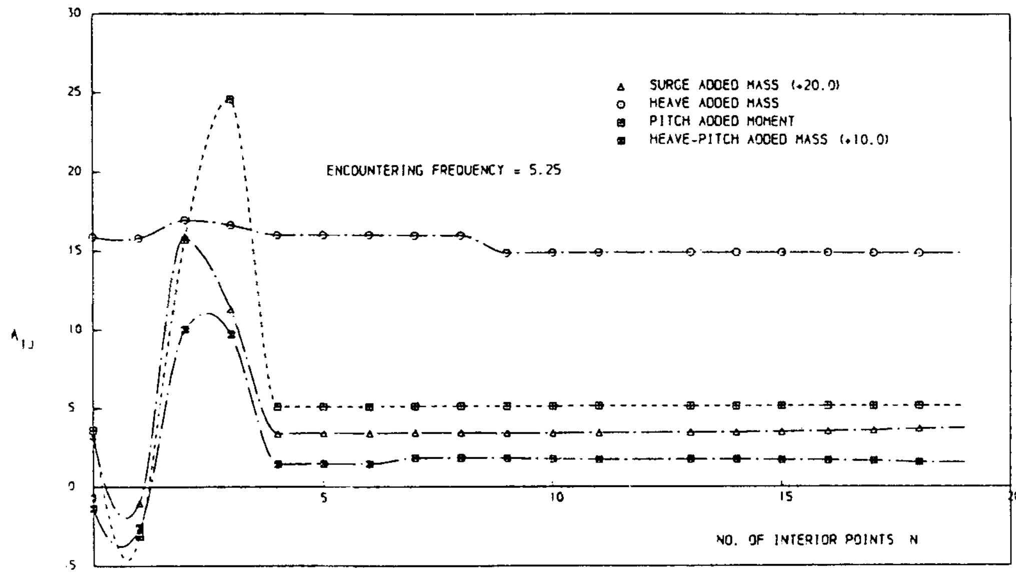

Figure 11. Influence of number of interior points upon hydrodynamic added mass coefficients

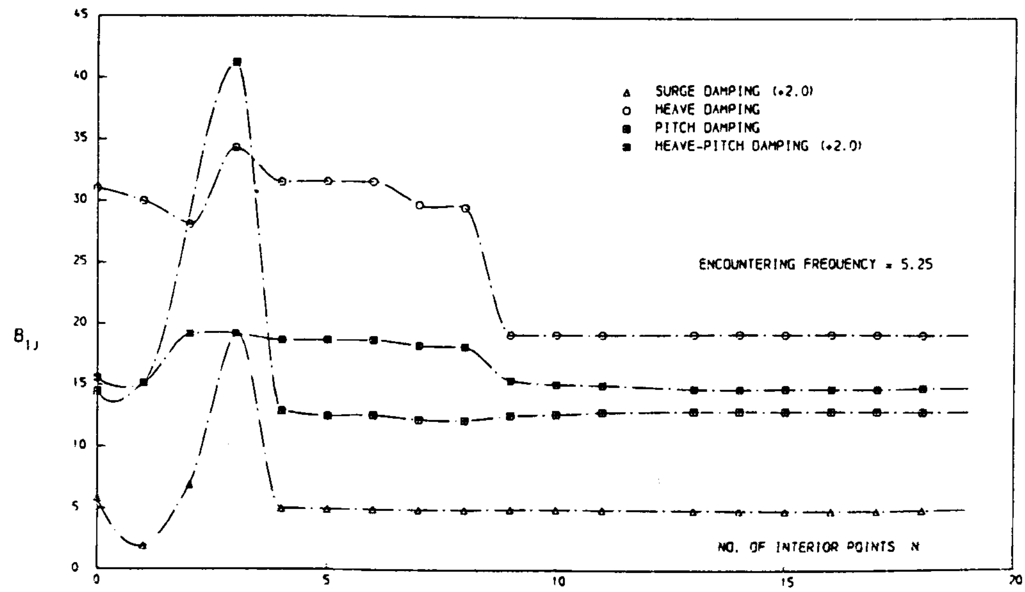

Figure 12. Influence of number of interior points upon hydrodynamic damping coefficients

relative CPU times incurred in the application of the three different integral equation formulations for a wave frequency of 5·25. The values presented have been normalized with respect to the CPU time of the direct method. Table I suggests that the CPU time of the CBIEM increases almost linearly with the number of interior points N'. When $N' = 20$ the CBIEM incurs a 16% penalty in CPU time over that of the direct method. It should be noted that these figures are only approximate since CPU time can vary by up to 5% depending upon the work load for the central processor in a time-sharing computer.

We have shown the effectiveness of the CBIEM at the first irregular frequency for the Series 60 model. To assess the performance of the methods in the higher-frequency range we extend the calculation up to and inclusive of the first eight irregular frequencies for a half submerged ellipsoid. The results are again compared with predictions based on the direct method of calculation. Figures 13 to 16 present the computed added mass and fluid damping coefficients using the two procedures. In the symmetrical mode of surge the CBIEM with 5 interior points can successfully eliminate the adverse effects of the first four irregular frequencies. The method breaks down at the fifth irregular frequency which occurs at a wave frequency of approximately 6·95. Increasing the number of interior points to 10 removes the difficulty. However, the surge added mass and damping coefficients exhibit some irregular patterns with the higher number of interior points. Nevertheless they show much improvement over that of the direct method.

For the antisymmetric sway mode the first irregular frequency occurs at a much higher frequency than that of the symmetric modes. In this case 5 interior points are not sufficient to eliminate its influence, whereas the use of 10 interior points improves the results substantially.

Table I. The normalized CPU time

	$N'=0$	4	8	12	16	20
Source method	0·98	—	—	—	—	—
Direct method	1·00	—	—	—	—	—
CBIEM	1·00	1·02	1·05	1·08	1·13	1·16

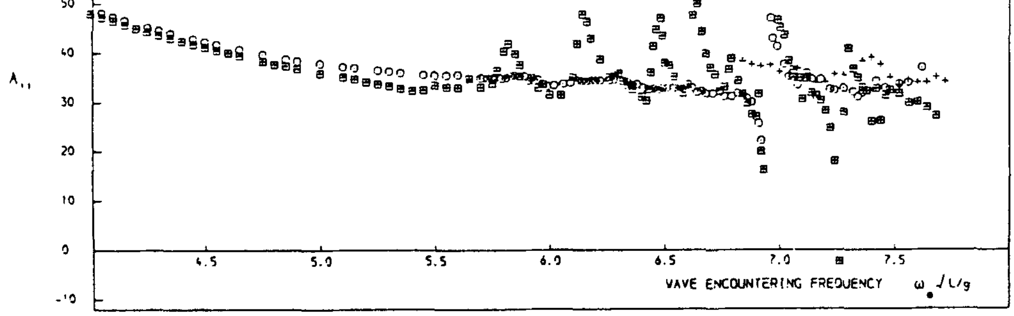

Figure 13. Surge added mass coefficient of an ellipsoid

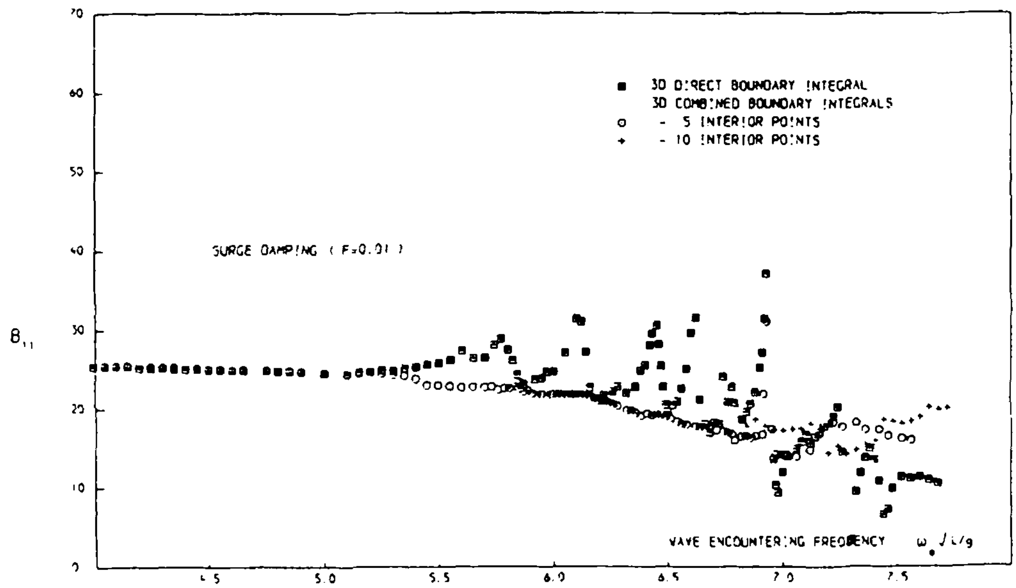

Figure 14. Surge damping coefficient of an ellipsoid

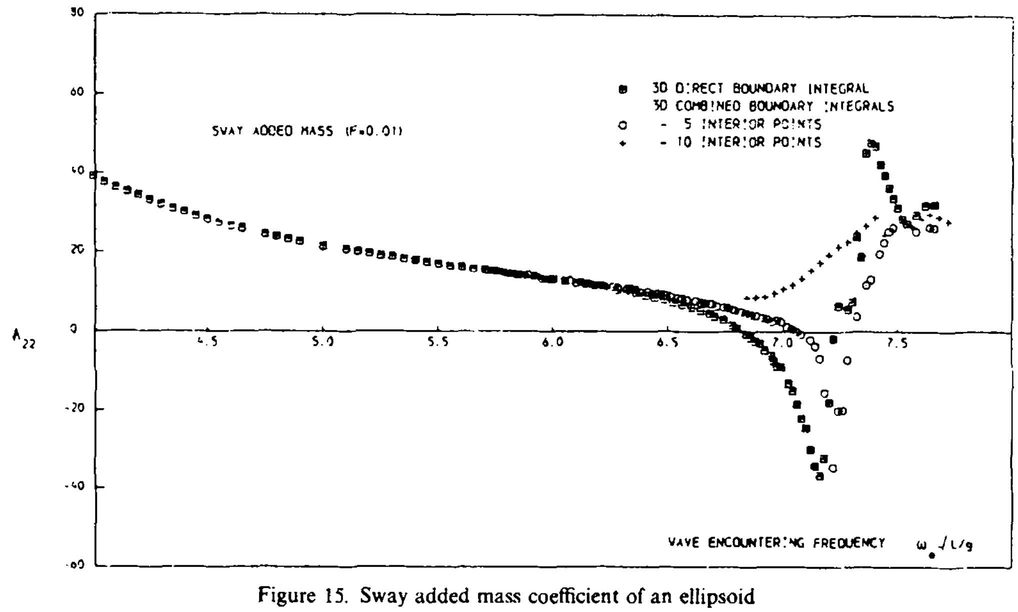

Figure 15. Sway added mass coefficient of an ellipsoid

5. FINAL COMMENTS AND CONCLUSIONS

The results presented are limited to the case of zero forward speed for which we have already proved the existence of the irregular frequencies associated with the integral equation. In the case of the forward speed problem little is known about the uniqueness of the integral equation. However, one can be sure that if irregular frequencies do exist in the forward speed problem, then their location will be quite different to those of the zero speed problem, simply because their

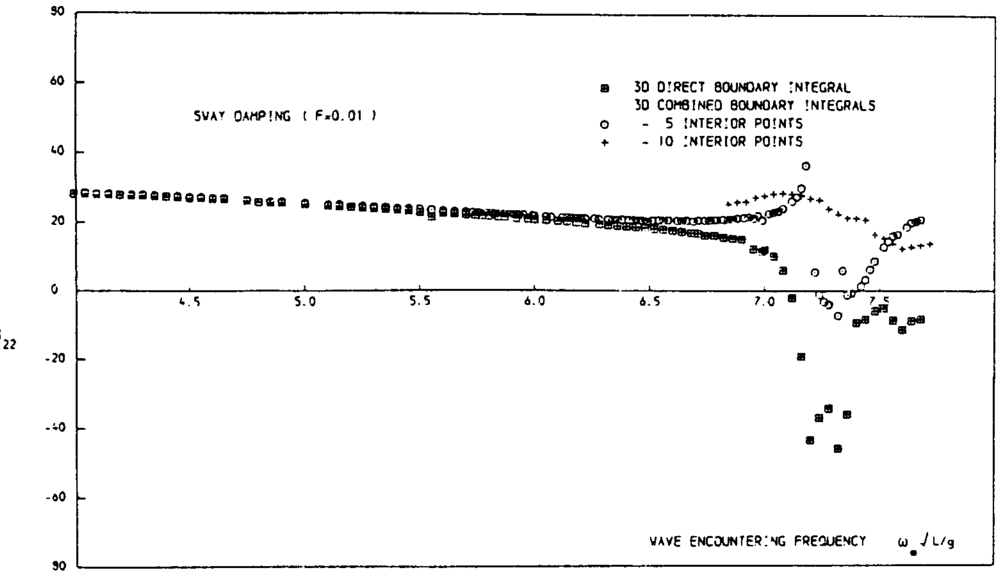

Figure 16. Sway damping coefficient of an ellipsoid

kernels are so different. Similarly, the location of the irregular frequencies in the forward speed problem formulated by the source integral and the direct Green function integral will also differ because the two integral equations are not adjoints, unlike the corresponding zero speed problem.

To summarize, we have demonstrated mathematically the existence of irregular frequencies in the integral equation associated with the water wave radiation and diffraction problem. These frequencies are connected to the eigensolution of the interior Dirichlet problem. The numerical difficulties related to the irregular frequencies can be eliminated by modifying either the domain of the integral operator or the integral operator on the same domain. The proposed combined boundary integral equation method (CBIEM) is effective in removing the first few irregular frequencies. The method does not require an excessive amount of extra computational effort and can easily be implemented in existing computer programs. However, at high frequencies the proposed procedure becomes less satisfactory as more interior points are needed. The computational penalty increases almost linearly with the number of interior points.

ACKNOWLEDGEMENT

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APPENDIX: PRINCIPAL DIMENSIONS OF HULL MODELS

Series-60 parent forms

Model number	4212W
Block coefficient, C_b	0·7
Length L (m)	3·048
Beam B (m)	0·435

Draught T (m)	0·174
Waterplane area A_w (m^2)	1·046
Volume of displacement (m^3)	0·162
Radius of gyration (m)	0·25L
Longitudinal centre of buoyancy (m)	F 0·015

Body lines are given by Todd.[25]

Ellipsoid

$$Y = B[(1 - X^2/L^2)(1 - Z^2/L^2)]^{1/2}$$

Length/beam ratio = 8
Beam/draught ratio = 2

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